This assignment has eight questions of total 80 marks. Answer all the questions. 20 marks are for viva voce. You may use illustrations and diagrams to enhance explanations. Please go through the guidelines regarding assignments given in the Programme Guide for the format of presentation. Illustrations/ examples, where-ever required, should be different from those given in the course material. You must use only simple calculator to perform the calculations.

1. (a) Consider that you are using the eight-decimal digit floating point representation as given in your Block 1, Unit 1, Section 1.3.1 page 29. Perform the following operations: *(3 Marks)*
   (i) Represent 5165485686 and 343456765 as floating point numbers using Rounding in normalised form.
   (ii) Given the above two numbers what is the absolute and relative error in their representation.
   (iii) Subtract the two numbers. What is the error in the resulting number?
   (iv) Divide the first number by the second number. Convert the result into normalized form in the given format.
   (v) Take the first number as 5165485686 and assume any second number to demonstrate the concepts of overflow and underflow for the given representation. (You may assume different second number and different operation for overflow and underflow).
   (vi) Define the concept of binary floating point numbers with the help of an example.

   (b) Consider the following two equations: *(2 Marks)*
   $x + y = 4$
   $0.499\ x + 0.501\ y = 2$
   Does the problem of solving the above two equations can be categorised as Ill-conditioned? Justify your answer.

   (c) Find the Maclaurin series for calculating cosine x. Use first four terms of this series to calculate the value of cosine of any value of x. Also find the bounds of truncation error for such cases. *(3 Marks)*

   (d) Obtain Approximate the value of $(0.99)^{-1}$ using first four terms of Taylor's series expansion. *(2 Marks)*

2. (a) Solve the system of equations *(5 Marks)*

$$2x + y + z = 3$$
$$x + 3y + 3z = 4$$
$$x - 4y + 2z = 9$$

using Gauss elimination method with **partial pivoting**. Show all the steps.

(b) Perform four iterations (rounded to four decimal places) using *(5 Marks)*
(i) Jacobi Method and
(ii) Gauss-Seidel method ,
for the following system of equations.

$$\begin{bmatrix} 5 & 4 & -3 \\ 4 & -4 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix}$$

With $\mathbf{x}^{(0)} = (0, 0, 0)^T$. The exact solution is $(1, -4, -5)^T$.

Which method gives better approximation to the exact solution?

3. Determine the smallest positive root of the following equation: *(10 Marks)*

$$f(x) \equiv x^3 - 9x^2 - x + 9 = 0$$

to three significant digits using
(a) Regula-falsi method (b) Newton-Raphson method (c) Bisection method (d) Secant method

4. (a) Find Lagrange's interpolating polynomial for the following data. Hence obtain the value of f(4). *(5 Marks)*

x	0	2	3	5
f(x)	2	11	21	121

(b) Using the inverse Lagrange's interpolation, find the value of x when y=3 for the following data: *(5 Marks)*

x	25	35	55	75
y=f(x)	-2	-1	1	5

5. (a) The population of a country for the last 25 years is given in the following table:. *(3+2+3 = 8 marks)*

Year (x) :	1995	2000	2005	2010	2015
Population in lakhs (y):	678	1205	1855	2745	3403

(i) Using Stirling's central difference formula, estimate the population for the year 2007
(ii) Using Newton's forward formula, estimate the population for the year 1998.

(iii) Using Newton's backward formula, estimate the population for the year 2013.

(b) Derive the relationship for the operators δ in terms of E. *(2 Marks)*

6. (a) Find the values of the first and second derivatives of y = f(x) for x=2.1 using the following table. Use forward difference method. Also, find Truncation Error (TE) and actual errors. *(5 Marks)*

x	:	2	2.5	3	3.5
y	:	8.7	12.7	16.8	20.9

(b) Find the values of the first and second derivatives of y = f(x) for x=2.1 from the following table using Lagrange's interpolation formula. Compare the results with (a) part above. *(5 Marks)*

x	:	2	2.5	3	3.5
y	:	8.7	12.7	16.8	20.9

7. Compute the value of the integral *(10 Marks)*

$$\int_0^8 (4x^4 + 5x^3 + 6x + 5)\, dx$$

By taking 8 equal subintervals using (a) Trapezoidal Rule and then (b) Simpson's 1/3 Rule. Compare the result with the actual value.

8. (a) Solve the Initial Value Problem, using Euler's Method for the differential Equation: *(4 Marks)*

$$y' = 1+x^2y, \text{ given that } y(0) = 1.$$

Find y(1.0) taking (i) h = 0.2 and then (ii) h = 0.1

(b) Solve the following Initial Value Problem using (i)R-K method of $O(h^2)$ and (ii) R-K method of $O(h^4)$ *(6 Marks)*

$$y' = xy + y^2 \text{ and } y(0) = 0.$$

Find y(0.4) taking h = 0.2, where y' means dy/dx